



Year 12-13 Transition Booklet

Pure Mathematics - Homework Book

Students Copy

Name _____
Teacher _____

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1. Surds and Indices

HOMEWORK MARK	SUBMISSION DATE	TARGET GRADE	HOMEWORK GRADE
SECTION B: SELF ASSESSMENT (to be completed by student) Please identify the areas in which you feel you have strengths and those in which you need to improve. Provide evidence to support yourself assessment with reference to the content of your homework.			
STRENGTHS		AREAS FOR IMPROVEMENT	
SECTION C: TUTOR FEEDBACK			
STRENGTHS		AREAS FOR IMPROVEMENT	

1. Write

$$\sqrt{75} - \sqrt{27}$$

in the form $k\sqrt{x}$, where k and x are integers.

(Total 2 marks)

2. (a) Find the value of $8^{\frac{4}{3}}$.

(2)

(b) Simplify $\frac{15x^{\frac{4}{3}}}{3x}$.

(2)

(Total 4 marks)

3. Simplify

(a) $(3\sqrt{7})^2$

(1)

(b) $(8 + \sqrt{5})(2 - \sqrt{5})$

(3)

(Total 4 marks)

4. (a) Expand and simplify $(7 + \sqrt{5})(3 - \sqrt{5})$.

(3)

(b) Express $\frac{7 + \sqrt{5}}{3 + \sqrt{5}}$ in the form $a + b\sqrt{5}$, where a and b are integers.

(3)

(Total 6 marks)

2. Completing the square

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Q3.

$$f(x) = x^2 + 4kx + (3+11k), \text{ where } k \text{ is a constant.}$$

Express $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found in terms of k .

(3)

(Total 3 marks)

A series of horizontal lines for writing.

3. Simultaneous Equations and Discriminant

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Q1. Solve the simultaneous equations

$$x + y = 2$$

$$4y^2 - x^2 = 11$$

(7)
(Total 7 marks)

Q2.

Solve the simultaneous equations

$$y - 3x + 2 = 0$$

$$y^2 - x - 6x^2 = 0$$

(7)

(Total 7 marks)

Q3.

Given the simultaneous equations

$$\begin{aligned}2x + y &= 1 \\ x^2 - 4ky + 5k &= 0\end{aligned}$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0 \tag{2}$$

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k . (3)

(c) For this value of k , find the solution of the simultaneous equations. (3)

(Total 8 marks)

A series of 25 horizontal lines, slightly slanted downwards from left to right, providing a template for handwriting practice.

4. Inequalities

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2. Find the set of values of x for which

(a) $3(x - 2) < 8 - 2x$

(2)

(b) $(2x - 7)(1 + x) < 0$

(3)

(c) both $3(x - 2) < 8 - 2x$ and $(2x - 7)(1 + x) < 0$

(1)

(Total 6 marks)

5. Curve Sketching

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1.

(a) Describe fully a single transformation that maps the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{3}{x}$. (2)

(b) Sketch the graph of $y = \frac{3}{x}$ and write down the equations of any asymptotes. (3)

2.

$$f(x) = 4x - 3x^2 - x^3.$$

(a) Fully factorise $4x - 3x^2 - x^3$. (3)

(b) Sketch the curve $y = f(x)$, showing the coordinates of any points of intersection with the coordinate axes. (3)

6. Functions, sketching & transformations

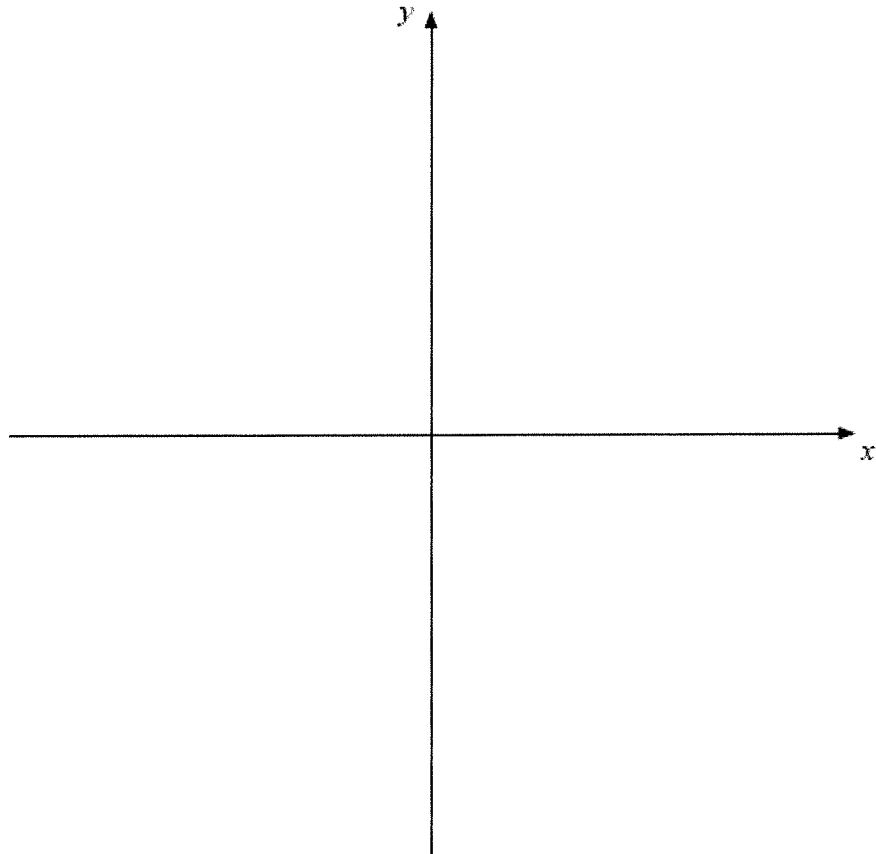
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1. (a) On the axes below sketch the graphs of

(i) $y = x(4 - x)$

(ii) $y = x^2(7 - x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.



(5)

(b) Show that the x -coordinates of the points of intersection of

$$y = x(4 - x) \text{ and } y = x^2(7 - x)$$

are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A , leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r and s are integers.

(7)

(Total 15 marks)

3. (a) Factorise completely $x^3 - 6x^2 + 9x$ (3)

(b) Sketch the curve with equation

$$y = x^3 - 6x^2 + 9x$$

showing the coordinates of the points at which the curve meets the x-axis. (4)

Using your answer to part (b), or otherwise,

(c) sketch, on a separate diagram, the curve with equation

$$y = (x - 2)^3 - 6(x - 2)^2 + 9(x - 2)$$

showing the coordinates of the points at which the curve meets the x-axis. (2)

(Total 9 marks)

7. Coordinate Geometry

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Q2.

The line l_1 has equation $3x + 5y - 2 = 0$

(a) Find the gradient of l_1 .

(2)

The line l_2 is perpendicular to l_1 and passes through the point $(3, 1)$.

(b) Find the equation of l_2 in the form $y = mx + c$, where m and c are constants.

(3)

(Total 5 marks)

Q3.

The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$,
where a, b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

(Total 7 marks)

Q4.

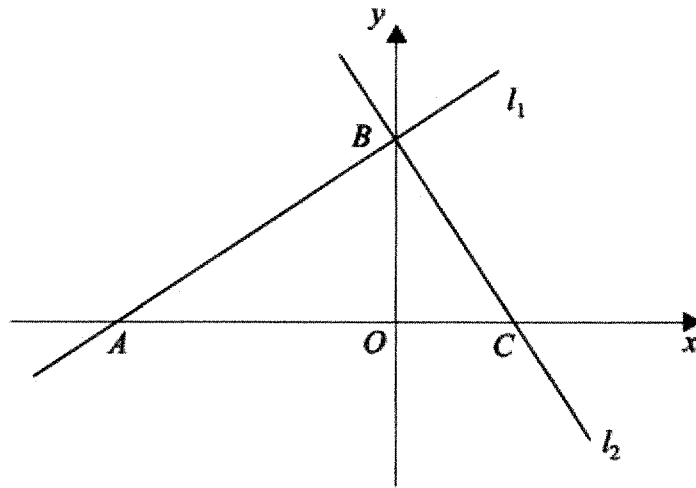


Figure 1

The line l_1 has equation $2x - 3y + 12 = 0$

(a) find the gradient of l_1 .

(1)

The line l_1 crosses the x -axis at the point A and the y -axis at the point B , as shown in Figure 1.

The line l_2 is perpendicular to l_1 and passes through B .

(b) Find an equation of l_2 .

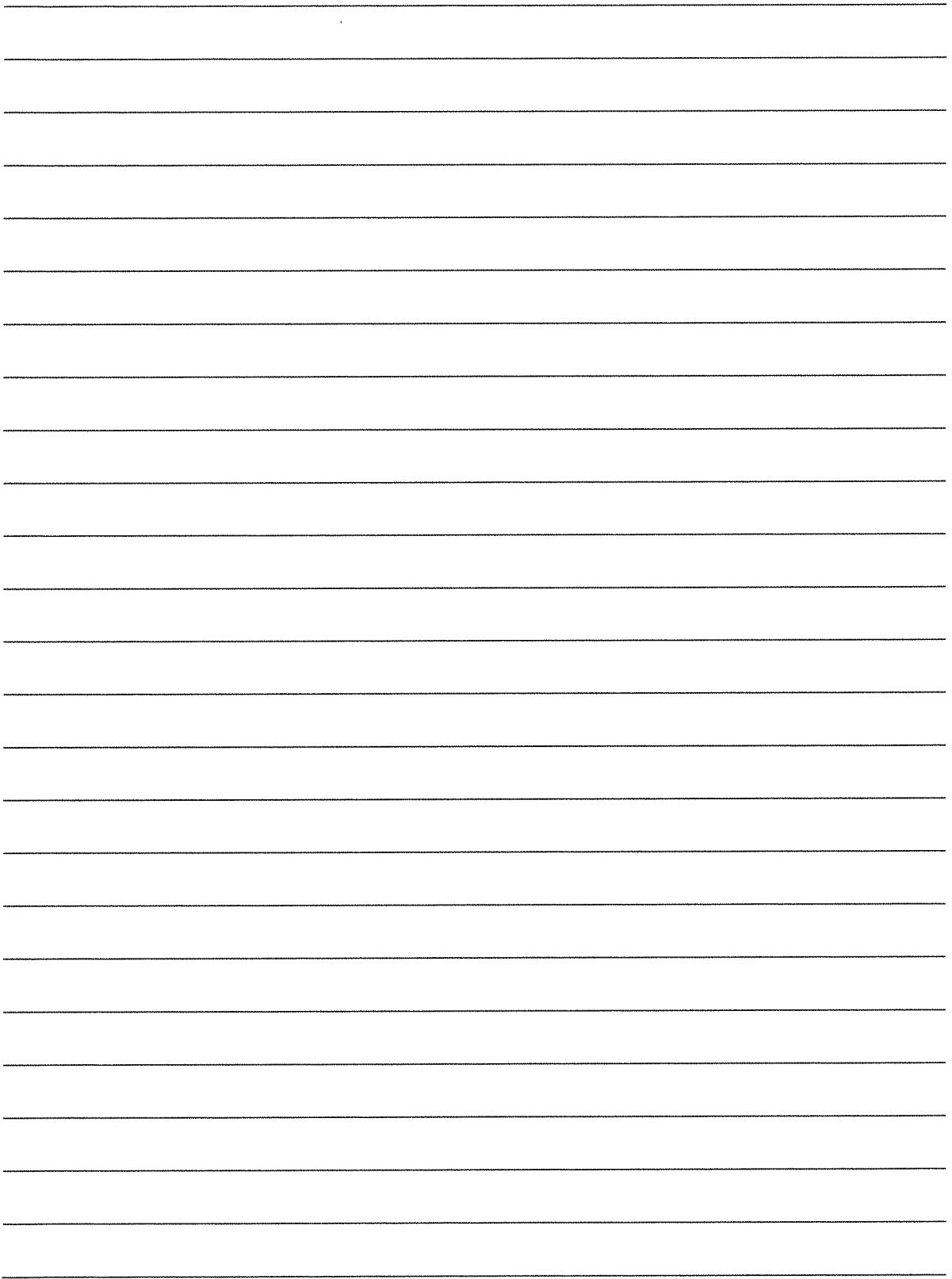
(3)

The line l_2 crosses the x -axis at the point C .

(c) Find the area of triangle ABC .

(4)

(Total 8 marks)



8. Sequences and Series

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1. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{a_n^2 + 3}, \quad n \geq 1,$$

$$a_1 = 2$$

(a) Find a_2 and a_3 , leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

(Total 4 marks)

2. A sequence x_1, x_2, x_3, \dots is defined by

$$x_1 = 1,$$

$$x_{n+1} = ax_n - 3, n > 1,$$

where a is a constant.

(a) Find an expression for x_2 in terms of a .

(1)

(b) Show that $x_3 = ax_2 - 3a - 3$.

(2)

Given that $x_3 = 7$,

(c) find the possible values of a .

(3)

(Total 6 marks)

4. A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d .

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d , (3)

(b) the value of a , (2)

(c) the total number of houses built in Oldtown over the 40-year period. (3)

(Total 8 marks)

Blank lined writing area consisting of 25 horizontal lines.

5. A sequence a_1, a_2, a_3, \dots , is defined by

$$a_1 = k,$$

$$a_{n+1} = 3a_n + 5, \quad n \geq 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k . (1)

(b) Show that $a_3 = 9k + 20$. (2)

(c) (i) Find $\sum_{r=1}^4 a_r$ in terms of k .

(ii) Show that $\sum_{r=1}^4 a_r$ is divisible by 10. (4)

(Total 7 marks)

6. The sum of an arithmetic series is

$$\sum_{r=1}^n (80 - 3r)$$

(a) Write down the first two terms of the series. (2)

(b) Find the common difference of the series. (1)

Given that $n = 50$,

(c) find the sum of the series. (3)
(Total 6 marks)

9. Differentiation

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1. Given that $y = x^4 + x^{\frac{1}{3}} + 3$, find $\frac{dy}{dx}$.

(Total 3 marks)

2. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

(Total 6 marks)

3. Given that $\frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$, can be written in the form $2x^p - x^q$,

(a) write down the value of p and the value of q .

(2)

Given that $y = 5x^2 - 3 + \frac{2x^2 - x^{\frac{3}{2}}}{\sqrt{x}}$,

(b) find $\frac{dy}{dx}$, simplifying the coefficient of each term.

(4)

(Total 6 marks)

5. The curve C has equation

$$y = x^3 - 2x^2 - x + 9, \quad x > 0$$

The point P has coordinates $(2, 7)$.

(a) Show that P lies on C . (1)

(b) Find the equation of the tangent to C at P , giving your answer in the form $y = mx + c$, where m and c are constants. (5)

The point Q also lies on C .

Given that the tangent to C at Q is perpendicular to the tangent to C at P ,

(c) show that the x -coordinate of Q is $\frac{1}{3}(2 + \sqrt{6})$. (5)

(Total 11 marks)

10. Integration

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SECTION C: TUTOR FEEDBACK			
STRENGTHS		AREAS FOR IMPROVEMENT	

1. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5) dx$$

giving each term in its simplest form.

(Total 4 marks)

2. Find $\int (3x^2 + 4x^5 - 7)dx$.

(Total 4 marks)

3.

$$\frac{dy}{dx} = 5x^{-\frac{1}{2}} + x\sqrt{x}, \quad x > 0$$

Given that $y = 35$ at $x = 4$, find y in terms of x , giving each term in its simplest form.

(Total 7 marks)

4. A curve has equation $y = f(x)$ and passes through the point $(4, 22)$.

Given that

$$f'(x) = 3x^2 - 3x^{\frac{1}{2}} - 7,$$

use integration to find $f(x)$, giving each term in its simplest form.

(Total 5 marks)

5. The curve C has equation $y = f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C, find

(a) $f(x)$, (5)

(b) an equation of the tangent to C at the point P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. (4)
(Total 9 marks)

A series of 20 horizontal lines for writing, evenly spaced and spanning most of the page width.

6. The curve C has equation $y = f(x)$, $x > 0$, and $f'(x) = 4x - 6\sqrt{x} + \frac{8}{x^2}$.

Given that the point $P(4, 1)$ lies on C ,

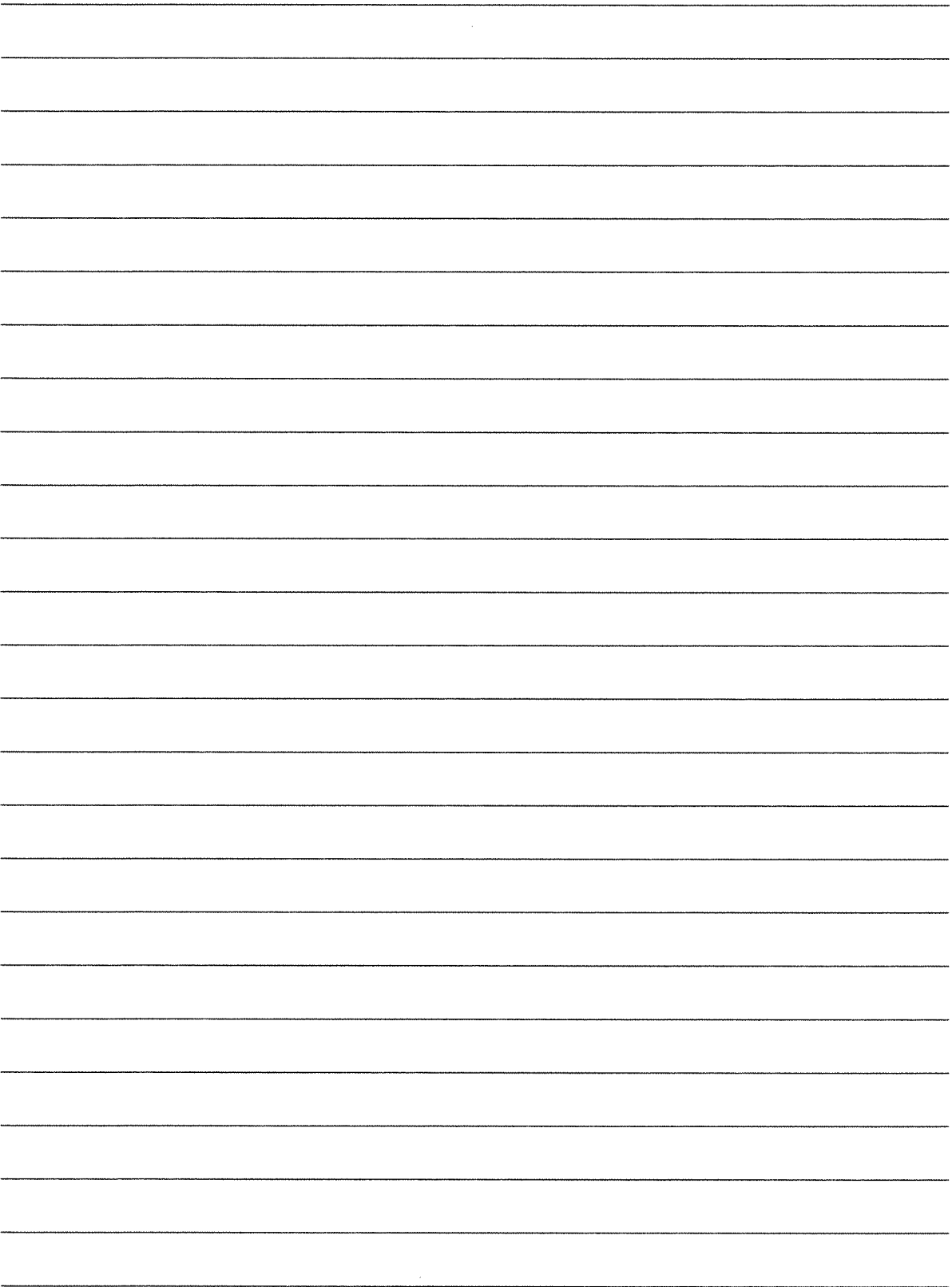
(a) find $f(x)$ and simplify your answer.

(6)

(b) Find an equation of the normal to C at the point $P(4, 1)$.

(4)

(Total 10 marks)



Topic: Partial Fractions

1. Express $f(x)$ in partial fractions when $f(x) = \frac{2x-1}{(x-1)(2x-3)}$ (3)

2. Express $\frac{3x+2}{x^2-2x-24}$ in partial fractions (4)

3. Find the values of the constants, A , B , and C in the identity (4)

$$3x^2 + 17x - 32 = A(x - 1)(x + 3) + B(x - 1)(x - 4) + C(x + 3)(x - 4)$$

4. Express in partial fractions $\frac{2x^2+4}{x(x-1)(x-4)}$

(4)

1. Find the value of A , B and C when,

$$2 - 9x = A(2x - 1)^2 + B(x - 3)(2x - 1) + C(x - 3)$$

(3)

2. Express $\frac{9x^2-2x-12}{x(x+3)(x-2)}$ in partial fractions

(4)

3. Express $\frac{13-3x^2}{(2x+3)(x-1)^2}$ in partial fractions

(4)
